Chapter 29 - Game Theory

→ Thus far we have focused on competitive equilibria
  → here, lots of agents were interacting and would take prices as given
  → there was no strategic interaction
  → we skipped chapters on monopoly and oligopoly, but will study strategic interaction in a general environment here

Games

→ A game is a strategic interaction between two or more economic agents (i.e. players)

→ Each player has a set of strategies
  → this set may be infinite, but we will usually focus on the case where it is finite.

→ The game is summarized by the players, their strategy sets, and their payoffs
  → payoffs are like utility from a certain outcome
    → "like" b/c slightly diff, we Neumann-Von Morgenstern utility to avoid thinking about risk aversion

→ The easiest way to represent a simultaneous game is with the normal form game:

- A normal form game looks like a table w/ rows and columns
- the rows correspond to player 1's strategies, the columns to player 2's strategies.
- in each cell are the payoffs to the players, given in a list (player 1 payoff, player 2 payoff)

→ Normal form:

		Player 2	
		Attack	Defend
Player 1	Attack	(2, 1)	(1, 0)
	Defend	(1, 2)	(0, 1)

strategies

payoffs
→ based on actions by each

How do we know the outcome (i.e. equilibrium) of the game?

→ the eq'm must be based on optimization - players try to maximize payoffs

→ the eq'm must be consistent - Player 1's actions are the ~~P~~ best response to Player 2's actions and Player 2's actions are the best response to Player's actions.

→ Easiest case - eliminate dominated strategies

→ Here we just take away any strategy a player ~~has~~ where the payoffs are less than other strategies for every strategy played by the other player(s).

e.g. game

		Player 2	
		Attack	Defend
Player 1	Attack	(2, 1)	(1, 0)
	Defend	(1, 2)	(0, 1)

if player 2 attacks

→ player 1 wants attack

if player 2 defends

→ player 1 wants to attack

⇒ attack is the dominant strategy for player 1

→ we can eliminate "defend"

→ she always wants to attack

if player 1 attacks

→ player 2 wants to attack

if player 1 defends

→ player 2 wants to attack

⇒ attack always dominant for player 2

⇒ we can eliminate "defend" for him also

⇒ eq'm in dominant strategies is (attack, attack)

Nash Equilibrium

→ Eliminating dominating strategies makes sense — who ~~?~~ would play a dominated strategy? - but is restrictive - how often are there only one dominant strategy for each player?

→ ~~Alternative~~ Alternative - Nash Eq'm

→ Nash Eq'm - Player 1 strategy best response to Player 2 strategy and Player 2 strategy best response to player 1

e.g. Prisoner Dilemma

Player 2

Player 1	Confess ~~coop~~	Deny ~~don't coop~~
confess ~~cooperate~~	-3, -3	0, -6
Deny ~~Don't~~ ~~coop~~	-6, 0	-1, -1

→ how find eq'm?

→ consider player 1

→ if Player 2 confess ~~coop~~, BR is ~~Don't~~ ~~coop~~ confess

→ if player 2 deny ~~don't~~, BR is ~~coop~~ confess

→ consider player 2

→ if player 1 confess ~~coop~~, BR is confess ~~coop~~

→ if player 1 deny ~~don't~~, BR is ~~deny~~ ~~don't~~ confess

→ Eq'm? (confess, confess) ~~(Don't, Don't)~~ only place where BR 1 + BR 2 intersect

→ This is called a Pure Strategy Nash Eq'm (PSNE)

→ Problems w/ PSNE

1) May not be one

2) May ~~be~~ more than one

3) Eq'm need not be Pareto optimal

→ e.g. Both prison's ~~[illegible]~~ off if ~~[illegible]~~ coordinate + both deny

Relaxing ~~P~~SNE — Mixed Strategy Nash Eq'm

→ if we allow players to randomize btwn strategies, their is more likelihood an eq'm exists

→ This is called an MSNE

→ At first this doesn't seem natural - why random?

→ But think about it and you see it is

→ In "rock, paper, scissors", do you always play rock?

→ In football, does a team who's ~~run~~ yards per carry exceed its yards per pass attempt always run the ball?

e.g. "Battle of the Sexes"

		Woman	
		Fight	Ballet
Man	Fight	(2, 1)	(0, 0)
	Ballet	(0, 0)	(1, 2)

→ 2 PSNE → find

→ 1 MSNE

→ how find?

Step 1 → if man mixing, he must be indiff. btwn F and B:

→ let p = prob woman chooses F

$u_m(F, p) = 2p + 0(1-p) = 2p$

$u_m(B, p) = 0p + 1(1-p) = 1-p$

$u_m(F, p) = u_m(B, p)$ → for what p?

$2p = 1-p$

$3p = 1$

$p = 1/3$ → if woman mixes w/ prob $p = 1/3$ then man will mix

→ How does man mix? use fact that woman only mixes if indiff btwn options

→ let q = prob man plays F

$u_w(F, q) = 1q + 0(1-q) = q$

$u_w(B, q) = 0q + 2(1-q) = 2-2q$

$\Rightarrow q = 2-2q$

$3q = 2$

$q = 2/3$

So unique MSNE is

$$((2/3, 1/3), (1/3, 2/3))$$

→ This randomization for man is the BR to this randomization for woman and the randomization for woman is the BR to this randomization for man.

Repeated games

→ Consider the Prisoner's Dilemma → they both confess is the unique PSNE
→ but the Pareto optimal is to both deny
→ why can't they cooperate and deny?
→ when might that happen?

→ one possibility is if game repeated

→ Suppose do 2 crimes together (and assume ~~know~~ get caught for both)
→ will they coop for first arrest so coop for second?

→ solve this by working backwards
→ 2nd arrest just like game we saw
⇒ (confess, confess) is PSNE
→ so confess here

→ now 1st arrest
→ why not confess? know will for 2nd
⇒ (confess, confess) PSNE ~~for~~ both arrests

→ only way can do better if game repeated indefinitely

→ then can have credible threat

e.g. suppose play trigger strategy:

→ Deny in every period unless someone has confessed in past

→ confess forever if someone has ~~played~~ confessed in the past

→ call this strategy s^*

→ let δ be the rate at which the players discount periods

Show can sustain cooperation where both deny

<u>step</u> 1: Show deny best resp. to deny

→ suppose no confession so far. Then given ~~coop deny~~ s^* played by other player, payoffs from confess and deny are:

$$\text{confess} = (1-\delta)\left[0 - 3\delta - 3\delta^2 - \dots\right]$$
$$= (1-\delta)\left(\frac{-3\delta}{1-\delta}\right) = -3\delta$$

$$\text{deny} = (1-\delta)\left[-1 - \delta - \delta^2 - \dots\right]$$
$$= \cancel{(1-\delta)}\left(\frac{-1}{\cancel{1-\delta}}\right)$$
$$= -1$$

⇒ deny better if:

$$-1 > -3\delta$$
$$+3\delta+1 \quad +3\delta+1$$
$$3\delta > 1$$
$$\delta > 1/3$$

<u>Step 2</u>: confess is BR to confess

→ suppose confession in past.
then according to s^*, the
other player always confesses
→ confess is BR to confess.

⇒ s^* an ~~eq.~~ Nash eq'm of
the repeated game
(technically a SPNE)

→ notice role of δ in <u>step 1</u>
→ if δ larger, can support more
diff. trigger strategies and
cooperative eq'm that is
further from (in terms of
payoffs) the one-shot PSNE

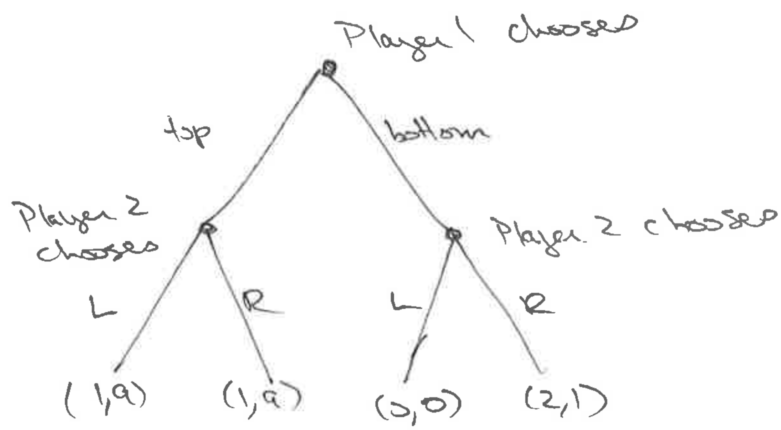
Sequential Games

→ Thus far we've done games where players act at same time

→ can also do sequential games

→ easiest to represent sequential games w/ the extensive form representation:

→ this looks like a decision ~~tree~~



→ analyze by working backwards

→ If Player 2 at top, what choose

→ L = R, indiff

→ If Player 2 at bottom, what choose

→ R > L ⇒ choose R
(1) (0)

→ so player 1 ~~...~~ know if choose

→ Top ⇒ get 1

→ Bottom ⇒ get 2, b/c #2 choose R

So eq'm:

~~B~~ #1 choose Bottom (2>1)

#2 choose R

→ Note how #2 wishes he could commit to L regardless → then #1 choose top & #2 get payoff of 9

→ Sequential games useful for thinking about deterrents to entry, commitments to strategies, etc.